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CRT-2002-28

Centre de recherche sur les transports (C.R.T.)
Centre for Research on Transportation

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September 2002

103-06394/

PUBLICATION
CRT-2002-28

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* This research was supported in part by CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nivel Superior), CNPq (Conselho Nacional de Desenvolvimento Científico e Technológico), two agencies of the Brazilian Federal Government, and by the Natural Science and Engineering Research Council of Canada (NSERC).

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Dépôt légal – Bibliothèque nationale du Québec
Bibliothèque nationale du Canada, 2002

ABSTRACT

This paper presents a spatial multi-product oligopoly model where the firms are subject to policy intervention on prices. This policy intervention is modeled as ceiling prices, and the equilibrium model is formulated as a variational inequality problem. Assuming the ceiling prices are high enough to remunerate the less efficient firm, in order no to exist an excess demand, a solution algorithm is proposed to determine the equilibrium supplies of each firm. The solution of this algorithm is compared with an extreme point solution obtained with a Linear Programming problem. A numerical analysis of the Brazilian petroleum refined products market is presented.

Key words: oligopoly equilibrium, variational inequalities, petroleum refineries.

RÉSUMÉ

Cet article présente un modèle d'oligopole spatial multi-produits où les sociétés sont assujetties aux politiques d'intervention des prix. La politique d'intervention est représentée comme un prix plafonné et le modèle correspondant est formulé comme une inégalité variationnelle. En supposant que les prix plafonnés sont assez élevés pour récompenser les sociétés moins efficaces, un algorithme de solution est proposé pour déterminer les offres à l'équilibre de chaque société. La solution obtenue avec cet algorithme est comparée avec une solution extrémale obtenue à l'aide d'un modèle de programmation linéaire. Un exemple numérique provenant du marché des produits pétroliers au Brésil est présenté.

Mots clés : équilibre d'oligopole, inégalité variationnelle, raffineries du pétrole.

1. Introduction

Price controls are common policy instruments used by governments in agricultural and energy programs. In agricultural markets, price floors at the supply markets are used to guarantee farmers a minimum price for their commodities. In the case of energy markets, price ceilings at the demand markets are used to ensure that prices for basic inputs for other industries are limited at certain level.

Market equilibrium models that usually handle such policy interventions consider a totally competitive market, where the firms are technologically similar, i.e., they have similar cost structures, and that they are small compared to the market. However, this assumption is often unrealistic, especially if the firms supplying the market are composed of producers, with large producing plants, and traders, which purchase products on the global market to sell locally. Petroleum refined products markets, an energy market, are examples of such non-totally competitive markets.

Oligopoly equilibrium models are more adequate for such markets, since they consider a finite number of firms trying to maximize their own profits. Each firm has perfect knowledge about the demand and it considers the strategies of the other firms when defining its own strategy. The main focus of this paper is to develop an oligopoly equilibrium model with policy interventions on prices.

In the next section, some spatial price equilibrium problems with policy intervention found in the literature are briefly reviewed. In section 3, oligopoly models are discussed and an oligopoly equilibrium model with policy intervention as price ceilings is proposed. An algorithm to determine the equilibrium supplies in the presence of price ceilings in an oligopoly market is described in section 4. Section 5 presents an application of this algorithm to the Brazilian petroleum refined products oligopoly market. Finally, the conclusions are summarized in section 6.

2. Spatial price equilibrium models with policy intervention

Nagurney (1993) presents a spatial price equilibrium formulation where there are policy interventions on prices as well as the possibility of excess supply and excess demand. Denoting the nonnegative possible excess supply at supply market $j \in J$ as u_j and the nonnegative possible excess demand at demand market $k \in K$ as v^k, and grouping them in column vectors **u** and **v**, the following feasibility conditions must hold:

$$s_j = \sum_{k \in K} y_j^k + u_j \text{ and } q^k = \sum_{j \in J} y_j^k + v^k, \text{ for all } j \text{ and } k, \qquad (1)$$

where s_j is the supply at supply market j (grouped in a vector **s**), q^k is the demand at demand market k (grouped in a vector **q**), and y_j^k is the shipment from j to k (grouped in a vector **y**).

In order to model policy interventions on prices, a fixed minimum supply price $\underline{\pi_j}$ (supply price floor) for each supply market j and a fixed maximum demand price $\overline{pr^k}$ (demand price ceiling)

at each demand market k are defined. Grouping them, one obtains the vectors $\underline{\boldsymbol{\pi}}$ and $\overline{\mathbf{pr}}$. π_j denotes the supply price at supply market j (grouped in a vector π), pr^k denotes the demand price at demand market k (grouped in a vector **pr**), and ct_j^k denotes the transshipment costs from j to k (grouped in a vector **ct**). Then the economic market conditions, assuming perfect competition, take the form: for all pairs of supply and demand markets $(j,k)\in(J,K)$:

$$\pi_j + ct_j^k \begin{cases} = pr^k, & \text{if } y_j^{k*} > 0 \\ \geq pr^k, & \text{if } y_j^{k*} = 0 \end{cases}, \quad \forall\ (j,k)\in(J,K), \tag{2}$$

$$\pi_j \begin{cases} = \pi_j, & \text{if } u_j^* > 0 \\ \geq \underline{\pi_j}, & \text{if } u_j^* = 0 \end{cases}, \quad \forall\ j \in J, \tag{3}$$

$$pr^k \begin{cases} = \overline{pr^k}, & \text{if } v^{k*} > 0 \\ \leq \overline{pr^k}, & \text{if } v^{k*} = 0 \end{cases}, \quad \forall\ k \in K. \tag{4}$$

Conditions (3) state that the supply price at each market j must be greater than or equal to the supply price floor, but if there is an excess supply then the supply price must be equal to the supply price floor. Analogously, (4) state that the demand price cannot exceed the demand price ceiling, and must be equal to the demand price ceiling if an excess demand exists in k.

In this case, the markets do not need to clear, i.e., positive profits for some firms as well as excess demand may occur. Then, the governing state may be one of disequilibrium. This problem can be formulated as a variational inequality problem: a pattern of total supplies, total demands, and commodity shipments, and excess supplies and excess demands (**s***,**q***,**y***,**u***,**v***), satisfying (1), satisfies inequalities (2), (3) and (4) governing the disequilibrium market problem if and only if it satisfies the variational inequality:

$$\boldsymbol{\pi}(\mathbf{s}^*)\cdot(\mathbf{s}-\mathbf{s}^*) - \underline{\boldsymbol{\pi}}\cdot(\mathbf{u}-\mathbf{u}^*) + \mathbf{ct}(\mathbf{y}^*)\cdot(\mathbf{y}-\mathbf{y}^*) - \mathbf{pr}(\mathbf{q}^*)\cdot(\mathbf{q}-\mathbf{q}^*) + \overline{\mathbf{pr}}\cdot(\mathbf{v}-\mathbf{v}^*) \geq 0, \tag{5}$$

for all (**s**,**q**,**y**,**u**,**v**) satisfying (1).

Nagurney and Zhao (1993) consider the spatial price equilibrium problem with price controls with direct demand functions. The problem is also formulated as a variational inequality problem. Basically, the price function **pr**(**d**) in (5) is replaced by a direct demand function.

3. Oligopoly Equilibrium with Policy Intervention

Instead of the totally competitive spatial price equilibrium model, consider now a spatial oligopoly equilibrium model, with firms that operate different technologies. One example of such a market is the petroleum refined products market, where the supply of these products can be made by producers, which operate refineries, and by traders, which purchase the products on the global market to sell to local markets. Policy interventions on prices in such markets are easy to be observed in practice. Tools to analyze the impact of price intervention in such oligopoly markets are of significant relevance.

The classical oligopoly model considers a demand market and a set F of firms supplying a homogenous product in a non-cooperative fashion. A Nash equilibrium (Nash, 1951) is established when all the firms set output levels in such a way that no firm alone can increase its own profit, producing more or less products. This problem is usually formulated as a variational inequality, considering the marginal profit of each firm. More detailed definitions of this model can be found in Murphy *et al* (1982), Harker (1984) and Nagurney (1993).

Harker (1986) and Nagurney (1993[2]) present a spatial oligopoly model where the profit function of each firm considers the transportation costs from the production plants to the demand markets. Harker (1986) compares the spatial oligopoly model with the totally competitive spatial price equilibrium and with two possible monopoly situations. All the four modeling approaches have a network structure, and can make use of network algorithms to find the equilibrium. As expected, in the examples presented, the oligopoly game profits, prices and flows are between those of the spatial price equilibrium and those of the monopoly model solution.

Okuguchi & Szidarovsky (1999) present a local (not spatially separated) oligopoly game with multi-product firms. Each firm has a cost function depending on the manufacturing of all products. In a general approach, a given limited input can be used to produce different products, then, the production levels of the products can depend on each other. However, when formulating the problem as a nonlinear complementarity problem, it is assumed that the production levels do not depend on each other and, for each product, they are in the interval [0, L^p], where L^p is the firm's production capacity for product p. Then, the set of all feasible output vectors X_f, for each firm f, equals the Cartesian product of the one-dimensional intervals [0, L^p].

Considering one product, the oligopoly equilibrium problem can be defined by a set F of firms trying to supply K demand markets, with separable demand price functions $pr^k(q^k)$ for each $k \in K$, where q^k is the quantity of the product demanded at market k; a set J of supply markets, where J_f is the set of supply markets where the firm f has production facilities (one facility for each supply market j), and each facility has the total cost function $C_{jf}(s_{jf})$, where s_{jf} is the quantity produced by firm f at supply market j; and average transportation cost functions $ct_j^k(y_j^k)$ for each pair (j,k), where y_j^k is the quantity shipped from j to k. Each firm f has a profit function of the form:

$$U_f = \sum_{k \in K} pr^k\left(q_f^k + Q_{F-f}^{k*}\right) q_f^k - \sum_{j \in J_f} C_j\left(s_{jf}\right) - \sum_{(j,k) \in (J_f, K)} ct_j^k\left(y_j^k\right) y_j^k . \qquad (6)$$

Q^{k*}_{F-f} is the optimal supply of all the other firms ($\neq f$) to the same market k. Thus, the profitability of one firm depends on the strategies (production levels) of the other firms. All the firms try to maximize their own profits when supplying the markets. A Nash-equilibrium is achieved when no firm unilaterally can improve its profit, changing its strategy.

Considering now a multi-product problem, where the price function of the product $p \in P$ may depend on the consumption levels of all products in all demand markets, $pr^{pk} = pr^{pk}(\mathbf{q})$, where the demand vector $\mathbf{q}$ has components q^{pk}, the quantity of product p demanded in market k from firm f. The production cost function of plant j of each firm $f \in F$ is now denoted by $C_{jf} = C_{jf}(\mathbf{s}_{jf})$, where $\mathbf{s}_{jf}$ is the output vector of all products from plant j. $\mathbf{s}$ is the output vector of all products from all the plants of all firms in F. The transportation costs depend on the entire shipment pattern, i.e.,

$ct_j^{pk} = ct_j^{pk}(\mathbf{y})$, where $\mathbf{y}$ has components y_{jf}^{pk}, the quantity of product p shipped to market k from plant j of firm f. Let Ω_{qsy} denote the production schedule constraints of each plant, and the flow conservation and non-negativity constraints. Hence, the equilibrium conditions of this oligopoly equilibrium model can be written as a variational inequality problem:

Find $(\mathbf{q}^*, \mathbf{s}^*, \mathbf{y}^*)$ such that

$$\sum_{f\in F}\sum_{j\in J_f}\sum_{p\in P}\frac{\partial C_{jf}(\mathbf{s}_{jf}^*)}{\partial s_{jf}^p}\left(s_{jf}^p - s_{jf}^{p*}\right) + \sum_{f\in F}\sum_{p\in P}\sum_{j\in J_f}\sum_{k\in K} ct_j^{pk}(\mathbf{y}^*)\left(y_{jf}^{pk} - y_{jf}^{pk*}\right)$$
$$- \sum_{f\in F}\sum_{p\in P}\sum_{k\in K} pr^{pk}(\mathbf{q}^*)\left(q_f^{pk} - q_f^{pk*}\right)$$
$$- \sum_{f\in F}\sum_{j\in J_f}\sum_{p\in P}\sum_{k\in K}\sum_{q\in P}\sum_{l\in K}\left[\frac{\partial pr^{ql}(\mathbf{q}^*)}{\partial q_f^{pk}} - \frac{\partial ct_j^{ql}(\mathbf{y}^*)}{\partial y_{jf}^{pk}}\right] y_{jf}^{ql}\left(y_{jf}^{pk} - y_{jf}^{pk*}\right) \geq 0, \ \forall(\mathbf{q}, \mathbf{s}, \mathbf{y}) \in \Omega_{qsy}. \qquad (7)$$

By using the flow conservation constraints (similar to (1)), it can be easily shown that the production cost functions $C_{jf}(\mathbf{s}_{jf})$, $\forall j$, and the price functions $\mathbf{pr}^{pk}(\mathbf{q})$, $\forall p$ and $\forall k$, are all dependent only on the transshipment quantities $\mathbf{y}$. Hence, the Kuhn-Tucker conditions for (7) are:

$$\begin{cases} \dfrac{\partial C_{jf}}{\partial s_{jf}^p} + ct_j^{pk} + \sum_{q\in P}\sum_{l\in K}\dfrac{\partial ct_j^{ql}}{\partial y_{jf}^{pk}} y_{jf}^{ql} \\ \qquad = pr^{pk} + \sum_{q\in P}\sum_{l\in K}\dfrac{\partial pr^{ql}}{\partial q_f^{pk}} q_f^{ql}, \text{ if } y_{jf}^{pk*} > 0 \\ \dfrac{\partial C_{jf}}{\partial s_{jf}^p} + ct_j^{pk} + \sum_{q\in P}\sum_{l\in K}\dfrac{\partial ct_j^{ql}}{\partial y_{jf}^{pk}} y_{jf}^{ql} \\ \qquad \geq pr^{pk} + \sum_{q\in P}\sum_{l\in K}\dfrac{\partial pr^{ql}}{\partial q_f^{pk}} q_f^{ql}, \text{ if } y_{jf}^{pk*} = 0 \end{cases}, \ \forall p\in P, \forall k\in K, \forall j\in J_f, \forall f\in F. \qquad (8)$$

Conditions (8) mean that a plant j of firm f will supply market k with product p if the derivative of the production cost in relation to the production of p in plant j, which is the marginal production cost of p, plus the marginal transportation costs to market k, $ct_j^{pk} + \sum_{q\in P}\sum_{l\in K}\dfrac{\partial ct_j^{ql}}{\partial y_{jf}^{pk}} y_{jf}^{ql}$, equals the marginal revenue, $pr^{pk} + \sum_{q\in P}\sum_{l\in K}\dfrac{\partial pr^{ql}}{\partial q_f^{pk}} q_f^{ql}$, of selling product p to market k.

If policy intervention using price ceilings is implemented, the following equilibrium conditions

$$pr^{pk} \begin{cases} = \overline{pr}^{pk}, & \text{if } v^{pk*} > 0 \\ \leq \overline{pr}^{pk}, & \text{if } v^{pk*} = 0 \end{cases}, \ \forall p\in P, \forall k\in K, \qquad (9)$$

similar to (4), must be added to the oligopoly equilibrium conditions (8), where v^{pk} denotes the excess demand of product p at demand market k, which can be grouped in a vector $\mathbf{v}$. The variational inequality formulation (7) now is rewritten as:

Find $(\mathbf{q}^*, \mathbf{s}^*, \mathbf{y}^*, \mathbf{v}^*)$ such that

$$\sum_{f\in F}\sum_{j\in J_f}\sum_{p\in P}\frac{\partial C_{jf}\left(\mathbf{s}_{jf}^*\right)}{\partial s_{jf}^p}\left(s_{jf}^p - s_{jf}^{p*}\right)+\sum_{f\in F}\sum_{p\in P}\sum_{j\in J_f}\sum_{k\in K} ct_j^{pk}\left(\mathbf{y}^*\right)\left(y_{jf}^{pk} - y_{jf}^{pk*}\right)$$

$$-\sum_{f\in F}\sum_{p\in P}\sum_{k\in K} pr^{pk}\left(\mathbf{q}^*\right)\left(q_f^{pk} - q_f^{pk*}\right)+\sum_{p\in P}\sum_{k\in K}\overline{pr}^{pk}\left(v^{pk} - v^{pk*}\right) \qquad (10)$$

$$-\sum_{f\in F}\sum_{j\in J_f}\sum_{p\in P}\sum_{k\in K}\sum_{q\in P}\sum_{l\in K}\left[\frac{\partial pr^{ql}\left(\mathbf{q}^*\right)}{\partial q_f^{pk}}-\frac{\partial ct_j^{ql}\left(\mathbf{y}^*\right)}{\partial y_{jf}^{pk}}\right] y_{jf}^{ql}\left(y_{jf}^{pk} - y_{jf}^{pk*}\right)\geq 0,\ \forall(\mathbf{q},\mathbf{s},\mathbf{y})\in \Omega_{qsy},\ \mathbf{v}\geq 0.$$

Excess demand for a product p in a demand market k will occur if the less efficient firm is not able to make a non-negative profit in selling to this market at the price ceiling. If for this less efficient firm the functions that make up the supply mapping are separable, i.e., there is no relation among the production of different products, this is the only way of an excess demand occurs. For example, in the petroleum refined products market, the supply of these products is usually made by producers, which operate refineries, and traders, which purchase the products in the global market and sell locally. The traders are usually less efficient than the producers. The production of refined products in refineries is not separable, since the quantity produced of one product is dependent of the quantity of the others. But for the traders, it is separable. They can trade only one kind of refined product if they want to. Hence, the petroleum refined products market is one example where the only way of excess demand occurs is when the less efficient firm cannot make a non-negative profit in selling at price ceiling.

This characteristic of the low possibility of occurrence of excess demand in such oligopoly markets will be used to develop solution algorithms to find equilibrium prices and supplies. Once the price ceilings are high enough to remunerate the less efficient firm, the equilibrium supplies will always be greater than or equal to the correspondent demands at the price ceilings.

4. Solution Algorithm

The first step in determining oligopoly equilibrium prices and supplies in the presence of price ceilings is to evaluate if these price ceilings are in effect. The equilibrium prices and supplies are computed considering no price ceilings. If the equilibrium prices are below the price ceilings, no need for future analysis is needed. However, it must not be the case.

Given that the price ceilings are really in effect, and that they are high enough to remunerate the less efficient firm in order to no excess demand occurs, how to determine the supply of each firm to each product and demand market at these price ceilings?

Assuming the price ceilings are fixed, a linear programming problem of the total joint profit of the industry can be formulated. The solution of this problem can be degenerate, since some markets may be equally profitable for two different firms. Hence, the solution of this problem may not be unique. This formulation cannot capture the competitive behavior of the firms in an oligopoly market. This gives the optimal supply pattern of a monopolistic firm, or even a

coalition of all the current firms in the market. For the case of non-cooperative oligopoly markets, such a linear formulation would probably not produce a realistic solution.

Oligopoly equilibration algorithms are usually based on diagonalization algorithms used to solve variational inequality problems (Harker, 1984, Pompermayer *et al*, 2002). They solve a profit maximization problem for each firm iteratively, assuming that all the other firms will not change their strategies (production levels), such as the profit function defined in (6). The strategy of each firm will affect the profit function in the others by changing the price functions. When price ceilings are in effect, one can define a minimum supply constraint for each market and product in the profit maximization problems solved in a diagonalization algorithm. However, these minimum supply constraints will depend on the initial supply of each firm defined at the initial iteration of the algorithm. This can make the convergence rate to a solution really small. The profitability of each firm tends to be small in the presence of the price ceilings and when the other firms are already supplying a great amount of the demand, it is hard for a firm find it profitable to sell more to this demand market and compete with the others. This is why such an approach has a low convergence rate and dependent on the initial solution.

In order to capture the non-cooperative nature of the firms in an oligopoly, and not depend on the initial solution, an oligopoly equilibration algorithm with adapted price functions is proposed, based on diagonalization algorithms used to find oligopoly equilibrium. This algorithm tries to find oligopoly equilibrium prices and supplies in such a way that the equilibrium prices equal the price ceilings. This is achieved by perturbing the slopes of the price functions, for each demand market and product, pivoting them at the points defined by the price ceilings and their respective "ceiling" demands. This model gives much more realistic supplies than the obtained with the linear problem of the total joint profit of the industry, as will be seen in the next section.

The final solution obtained with this model is also not dependent on the initial solution. Once the price functions are now more horizontal than the original ones, one firm has more flexibility in producing more units of a given product, when maximizing its profit, i.e., it can still make a positive profit. This extra supply will not affect so much the prices as if the original price function would be used. Also, there is no need to define an explicit constraint of minimum demand in each profit maximization problem, since the oligopoly equilibrium supplies will equal the demand which correspond to the price ceilings. This algorithm is summarized below for the case of linear price functions of the form

$$pr^{pk}\left(d^{pk}\right) = \frac{pr_0^{pk}}{\varepsilon_k^{pp}} \cdot \frac{d^{pk}}{d_0^{pk}} + \frac{pr_0^{pk}\left(\varepsilon_k^{pp} - 1 - \Delta pr^{q \neq p,k}\right)}{\varepsilon_k^{pp}}, \tag{11}$$

for each product p and demand market k, which are dependent only on the total demand d^{pk} for the product p at demand market k. ε_k^{pp} is the price elasticity of product p at demand market k. pr_0^{pk} and d_0^{pk} are known price and demand for product p at demand market k, in a given period of time. The term $\Delta pr^{q \neq p,k}$ captures cross price effects between different products, using the prices obtained in the previous iteration of the algorithm for oligopoly equilibration and the cross price elasticities ε_k^{pq}. It is defined as:

$$\Delta pr^{q \neq p,k} = \sum_{q \in P, q \neq p} \varepsilon_k^{pq} \frac{\left(pr^{qk} - pr_0^{qk}\right)}{pr_0^{qk}}. \tag{12}$$

The adapted price functions differ from the original ones by the price elasticities ε_k^{pp} for each product p at each demand market k.

Fixed Price Oligopoly Solution Algorithm:

Step 1 (*Initialization*): Solve the oligopoly equilibrium problem without considering the price ceilings. Let $pr^{pk*}(0)$ be the equilibrium price of product p at demand market k and $\varepsilon_k^{pp}(0)$ the original price elasticity for p and k. Set t=1.

Step 2 (*Iteration*): For each p and k where $pr^{pk*}(0) > \overline{pr}^{pk}$, adapt the price function $pr^{pk}(.)$, by changing the price elasticity as

$$\varepsilon_k^{pp}(t) = \varepsilon_k^{pp}(t-1)\left(1 + \frac{pr^{pk*}(t-1) - \overline{pr}^{pk}}{\overline{pr}^{pk}}\right),$$

and find new equilibrium prices $pr^{pk*}(t)$ solving the oligopoly equilibrium problem with these adapted price functions.

Step 3 (*Convergence test*): If for all p and k, $(pr^{pk*}(t) - \overline{pr}^{pk}) < \epsilon$, some small tolerance, stop. Otherwise, set $t = t + 1$, and go to step 1.

Note that it is assumed that the original price functions $pr^{pk}(.)$ are decreasing with the quantity of the product p demanded in market k, i.e., $\varepsilon_k^{pp} < 0$.

5. A Numerical Example

The algorithm presented in the previous section was applied to the Brazilian market of petroleum-refined products. The LP problem of the total joint profit of the industry is also analyzed. This application is based on the data available for the month of April 2001. Price elasticities were estimated based on the time series data from 1977 to 1997. Linear price functions are fitted to data, which include cross price effects.

Petroleum refined products are usually produced in refineries. The profit maximization problem of refining companies is not simple. It must consider the special characteristics of the refining process. The production of each product is closely related to the production of the others. The quantity that will be produced of each product depends on the crude oil characteristics, the complexity of the refinery (i.e., the ability to transform heavy hydrocarbon chains into lighter ones), and on the operating mode chosen. Petroleum refining companies usually make use of linear programming models to optimize the operation of the refineries and prepare the production schedules. Then, the profit maximization problem of refining companies must consider this cost minimization problem formulated as a linear programming problem. More detailed description about this profit maximization problem is given in Pompermayer *et al* (2002).

Three aggregated products are considered: light distillates, medium distillates and heavy distillates[1]. Table 1 shows the price and cross price elasticities for these three products. There are fifteen internal demand sub-regions where the demand functions for these products were

[1] The main product in light distillates is gasoline, diesel oil in the medium distillates, and fuel oil in the heavy distillates.

calibrated. These sub-regions have been defined to correspond to storage terminals and distribution centers used by distribution companies, where the competition in supplying refined products is expected to occur.

Table 1: Price elasticities used in the simulation.

Price elasticities	Gasoline	Diesel	Fuel oil
Gasoline	-0.15	0.03	0.00
Diesel	0.05	-0.12	0.04
Fuel oil	0.00	0.03	-0.40

Three refining companies can supply any of the internal demand sub-regions, as well as export to an external sub-region. Transportation costs to any of the sub-regions are evaluated by considering the mode used, and the capacity of each mode and terminal, in the case of multi-modal transportation. Two of these companies operate one refinery each, and buy crude oil on the global market. These are labeled Refiner 2 and Refiner 3 in the Tables below. The other company, the larger one (labeled Refiner 1), operates 10 refineries and several crude oil production facilities. It supplies more than 90% of the existing refining capacity in Brazil. Figure 1 shows a map of Brazil with the refineries, terminals and the demand sub-regions considered in this example.

Figure 1: Brazil, demand sub-regions, refineries and terminals.

Seven crude oil types are considered: three produced in Brazil, and four from other countries that usually trade crude oil with Brazil. Acquisition costs for the "foreign" crude oils are estimated based on prices published on the market and on the transportation costs to each refinery. Crude oil production costs were estimated by considering the type of exploitation (on-shore / off-shore).

Each refinery may operate in two modes: "maximum gasoline" and "maximum diesel".

Therefore, conversion rates from each crude oil into each refined product were estimated for each refinery by using a given operating mode. This estimation considered the complexity of each refinery and the characteristics of the crude oil, especially density. It was used in estimating refining costs as well.

It is assumed that one trader may also supply refined products to any of the 15 internal demand regions, acquiring products basically in two foreign points, Argentina and Venezuela. The trader purchases the products at a fixed price, i.e., no change in prices occurs if the quantity purchased increases. This assumption is based on the fact that a trader must be small compared to the global market, not influencing the total demand in the acquisition points unilaterally. However, if the supplying capacity of the points where the trader can purchase products is not considerably big, supply price functions, increasing with the quantity purchased, must be evaluated. It is also assumed that transportation costs are constants.

The price ceilings, for all 15 internal sub-regions, are \$650,00/m^3 for light distillates, \$600,00/m^3 for medium distillates and \$550,00/m^3 for heavy distillates, which are high enough to remunerate the trader, the less efficient firm. For the external sub-region no price ceiling is defined since the prices are assumed constant at this sub-region. Table 2,3 and 4 below shows the supplies for light, medium and heavy distillates, respectively, of each firm in the oligopoly with these price ceilings. It is also shown the equilibrium prices if no policy intervention were in effect, obtained at the iteration zero of the algorithm presented in the previous section. The supplies are then compared to the supplies obtained with the LP problem of the total joint profit of the industry. This total joint profit is greater than the sum of the profit of each firm obtained in the first algorithm.

Table 2: Light distillates supplies [m^3].

	Eq. prices	Price Ceiling Oligopoly Equilibrium				LP Total Joint Profit			
Sub-regions	[\$/m^3]	Trader	Refiner 1	Refiner 2	Refiner 3	Trader	Refiner 1	Refiner 2	Refiner 3
PA/AP	1,568.50	11,141	15,999	0	0	0	27,240	0	0
AM/RR/RO/AC	1,544.90	13,162	22,081	0	0	0	35,356	0	0
MA/PI	1,563.04	10,818	14,733	0	0	0	25,634	0	0
CE/RN	1,556.84	21,157	30,821	0	0	0	52,163	0	0
PE/PB/AL	1,559.43	29,941	42,578	0	0	0	72,736	0	0
BA/SE	1,526.87	28,311	50,896	0	0	0	79,408	0	0
RJ/ES	1,512.49	59,258	103,775	2,354	10,906	0	147,648	0	28,980
MG	1,511.08	60,098	113,062	0	7,632	0	181,286	0	0
SP W	1,514.30	45,496	91,532	263	1,323	0	139,008	0	0
SP central	1,520.64	63,294	124,887	1,990	2,220	0	192,913	0	0
SP NE	1,517.43	96,706	178,528	5,712	7,846	0	289,369	0	0
RS	1,500.12	57,495	81,713	6,444	3,042	0	129,709	18,986	0
PR/SC	1,500.39	79,829	113,545	2,225	4,343	0	200,049	0	0
MS/MT	1,719.96	10,633	35,345	0	0	0	46,800	0	0
GO/DF/TO	1,574.56	35,726	79,314	0	0	0	115,497	0	0
Sub-total	-	623,064	1,098,807	18,986	37,312	0	1,734,816	18,986	28,980
External	414.80	-	1,006,739	0	0	0	114,291	0	0
Profit [10^6\$]	-	255.271	2,069.423	15.512	18.282	2,512.214			

The equilibrium prices in the oligopoly without price ceilings are higher than the price ceilings for all internal sub-regions. The LP problem of the total joint profit of the industry gives an extreme and degenerate solution. The degeneracy occurs at the sub-regions 'RJ/ES' and 'RS', coincidently the sub-regions where the small refiners locate their refineries. The Trader has a zero supply because the refiners have enough production capacity to supply all the internal demand. This solution is not realistic if the firms are not cooperative.

The results of the "Fixed Price Oligopoly Solution Algorithm" using the price ceilings are more realistic, capturing the competition among the firms in a non-cooperative fashion. The four firms compete in practically all the sub-regions. The small refiners do not supply all the sub-regions given their small production capacities. The sum of the profit of the four firms is smaller than the joint profit obtained with the LP problem, as expected.

Table 3: Medium distillates supplies [m^3].

Sub-regions	Eq. prices [$/$m^3$]	Price Ceiling Oligopoly Equilibrium				LP Total Joint Profit			
		Trader	Refiner 1	Refiner 2	Refiner 3	Trader	Refiner 1	Refiner 2	Refiner 3
PA/AP	1,649.19	33,110	53,839	0	0	87,378	0	0	0
AM/RR/RO/AC	1,625.51	33,786	64,152	0	0	51,405	47,199	0	0
MA/PI	1,626.03	28,298	39,161	0	0	67,695	0	0	0
CE/RN	1,635.05	23,755	43,413	0	0	0	67,544	0	0
PE/PB/AL	1,642.62	40,093	61,302	0	0	101,748	0	0	0
BA/SE	1,623.12	59,984	129,709	0	0	0	190,311	0	0
RJ/ES	1,600.73	79,690	149,891	868	2,475	0	214,180	0	12,679
MG	1,600.04	84,923	183,206	0	0	0	260,974	0	0
SP W	1,602.46	89,034	201,410	0	0	0	282,456	0	0
SP central	1,604.56	65,586	145,066	0	0	0	204,897	0	0
SP NE	1,601.39	101,544	209,221	3,293	0	0	305,543	0	0
RS	1,588.23	77,427	157,474	11,061	0	0	222,642	16,817	0
PR/SC	1,599.35	140,317	268,683	1,594	0	0	400,179	0	0
MS/MT	1,845.00	37,193	172,274	0	0	0	214,553	0	0
GO/DF/TO	1,674.48	47,974	122,991	0	0	0	171,758	0	0
Sub-total	-	942,712	2,001,791	16,817	2,475	308,226	2,582,237	16,817	12,679
External	384.60	-	195,490	0	0	-	0	0	0

The refiners do not have enough production capacity to supply all the demand for medium distillates, then, in the LP solution, the Trader could supply some amount of these products to the farer (from the refineries) sub-regions. The results of the "Fixed Price Oligopoly Solution Algorithm" are similar to the ones obtained for light distillates.

Table 4: Heavy distillates supplies [m^3].

Sub-regions	Eq. prices [$/m^3]	Price Ceiling Oligopoly Equilibrium				LP Total Joint Profit			
		Trader	Refiner 1	Refiner 2	Refiner 3	Trader	Refiner 1	Refiner 2	Refiner 3
PA/AP	569.11	13,645	20,306	0	0	0	34,054	0	0
AM/RR/RO/AC	565.98	8,043	13,740	0	0	0	21,810	0	0
MA/PI	561.73	2,620	3,452	0	7	0	6,094	0	0
CE/RN	570.38	513	831	7	11	0	1,367	0	0
PE/PB/AL	559.38	2,081	2,977	83	113	0	5,266	0	0
BA/SE	558.02	4,515	8,402	305	338	0	13,585	0	0
RJ/ES	538.66	18,400	34,026	1,948	3,802	0	45,008	0	12,075
MG	545.74	23,556	48,163	1,647	4,140	0	76,796	0	0
SP W	545.48	19,180	38,753	2,153	1,687	0	61,410	0	0
SP central	545.05	17,296	34,337	2,334	1,583	0	55,169	0	0
SP NE	541.97	22,136	41,825	3,203	2,679	0	68,962	0	0
RS	530.47	8,241	12,866	1,190	324	0	7,766	14,104	0
PR/SC	534.42	13,811	21,516	938	808	0	36,099	0	0
MS/MT	610.55	1,028	2,913	0	0	0	3,959	0	0
GO/DF/TO	566.96	8,108	16,937	296	84	0	25,506	0	0
Sub-total	-	163,173	301,045	14,104	15,577	0	462,852	14,104	12,075
External	280.00	-	1,185,024	0	0	-	828,754	0	0

For heavy distillates the price ceiling of $550.00 is not in effect at all internal sub-regions, as the equilibrium prices without policy intervention indicate. The "Fixed Price Oligopoly Solution Algorithm" managed well the difference among the sub-regions with and without price ceilings.

6. Conclusions

A multi-product spatial oligopoly equilibrium problem with policy intervention using price ceilings was formulated as a variational inequality problem. Excess demand will occurs if the price ceilings are not sufficiently high to cover the costs of the less efficient firm in the oligopoly market. This characteristic was exploited to develop an algorithm to find the market share of each firm given the price ceilings. A linear problem of the total joint profit of the industry was also presented in order to show that this formulation describes a monopolistic firm or a coalition of the firms in the market, i.e., not a non-cooperative oligopoly market.

An application to the Brazilian petroleum refined-products market was presented. This shown the possibility of solving large problems where the profit maximization problems of each firm are complex and the production of one product is dependent on the others.

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